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博士学位论文

随机动力系统的概率密度演化
—数值方法与扩展

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The Probability Density Evolution of Stochastic Dynamical Systems - The Numerical Method and Extension

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摘要

实际工程结构在其服役期间不可避免地存在各种随机因素的影响，其在外部作用下的响应和性态也表现出复杂的随机特性。针对这一问题，近年来，李杰和陈建兵发展了一类概率密度演化方法（PDEM），利用这一方法可以方便地求解各类随机动力系统的概率密度演化进程。然而，对于宽带白噪声激励下随机动力系统响应的求解和广义密度演化方程的数值求解方法，虽然开展了一定的研究，但仍存在着诸多的不足，需要更进一步的发展。本文正是从这一角度出发，对随机动力系统的概率密度演化理论做出了进一步的深入研究。

对于高斯白噪声或者过滤白噪声激励，现有的谱表现方法需要采用数百个随机变量才能对其良好表述。建立由含有限个随机变量的随机函数模型表达宽带白噪声激励，有助于推广概率密度演化方法在随机动力系统中的应用。本文在陈建兵和李杰提出的一类随机谐和函数的基础上，给出了一类新的随机谐和函数模型，并发现了两类随机谐和函数的共性。从理论上证明：这类模型具有渐进正态性，可以以较少的随机谐和分量合成具有较丰富概率信息的随机过程。以单自由度系统为例，详细分析了其在第二类随机谐和函数激励下的响应，并提出了频率区间优化方法。最后，以多自由度体系为例验证了有效性。

采用概率密度演化方法求解随机动力系统，相比于随机模拟方法，需要的确定性动力响应分析工作量大大降低。然而，在实际应用当中，确定性动力系统代表性时程计算仍会耗费大量的时间。利用质点近似和再生核质点近似，本文发展了随机动力系统的再生核质点加密算法。采用这一方法，在几乎不增加计算时间的基础上，显著提高了随机动力系统响应概率密度解答的精度。

概率密度演化的数值算法，虽然经过了十多年的发展，仍采用以差分算法为主的计算方法。利用差分方法求解广义概率密度演化方程，目前所选用的格式尚存在一些问题。本文建立了广义概率密度演化方程求解的再生核配点法，并进行了稳定性分析。通过线性单自由度系统和多自由度框架结构的求解，证实这一方法的解答相比与现阶段采用的 TVD 差分格式更具优越性。

最后总结本文工作，对进一步研究工作做出了展望。

关键词：随机动力系统，随机谐和函数，广义概率密度演化方程，再生核质点加密算法，再生核配点法

ABSTRACT

Engineering structures in service will unavoidably encounter various categories of uncertainties. As a result, the response of structure under random excitation also exhibits randomness. Recently. Li Jie and Chen Jianbing has been developed a class of probability density evolution method (PDEM). The instantaneous PDF of the response of stochastic dynamic systems and its evolution is available by PDEM. For the last few years, efforts have been made on two aspects: (1) the response of stochastic dynamic systems under the excitation of white noise; (2) the numerical methods of probability density evolution equation (PDEE). There are still many shortage of them. The PDEM of stochastic dynamic system is studied in depth.

The spectrum representation method is commonly used for Gaussian white noise or colored Gaussian noise. This method demands hundreds of random variables, making the solution of stochastic dynamics very cumbersome. Reducing the number of stochastic variables will greatly facilitate the application of PDEM in stochastic dynamic systems. Based on the previous work on the stochastic harmonic functions of the first kind, a new stochastic harmonic function was developed in this thesis. The similarities between the two kinds of the stochastic harmonic functions were also identified. The power spectral density of the process represented by the stochastic harmonic function of the second kind was proven identical to the target power spectral density. It was also proven in theory that the process represented by the stochastic harmonic function of the second kind is asymptotically Gaussian. Taking the single degree of freedom system into account, the response of the stochastic harmonic function of the second kind was investigated and the optimal selection of frequencies was also suggested. Finally, linear and nonlinear responses of a multi-degree-of-freedom system subjected to random ground motions were analyzed to demonstrate the effectiveness and superiorities of the proposed approach.

Comparing to the Monte Carlo method, the computational cost of deterministic dynamic response analysis is reduced dramatically by the PDEM. However, the computation of the deterministic response analysis costly. Based on the particle approximation and reproducing kernel particle method, this thesis developed the refining algorithm of stochastic dynamic system. This method significantly improves the accuracy of the PDF and its evolution of the response of stochastic dynamic system without adding computational costs.

Finite difference has been used to solve PDEE for decades, which is far from optimal. To improve on the numerical method for PDEE, this thesis developed the reproducing kernel collocation method (RKCM). The stability of this method was also analyzed. The performance of RKCM was superior over the TVD scheme, as revealed by the applications on single degree of freedom system and the multi degree of freedom structures.

Finnaly, the main works of the thesis are summarized and the prospect of future work is proposed.

Key words: Stochastic dynamic systems, Stochastic harmonic function, Generalized probability density evolution equation, Reproducing kernel particle refining algorithm, Reproducing kernel collocation method

目录